

HW Mark: 10 9 8 7 6 RE-Submit

Systems of Linear Equations

This booklet belongs to: _____ Period ____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
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		Pg.	
		REVIEW	
		TEST	

Your teacher has important instructions for you to write down below.

Systems of Linear Equations

STRAND Relations & Functions		DAILY TOPIC	EXAMPLE
C9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically	9.1	Model a situation, using a system of linear equations.	
	9.2	Relate a system of linear equations to the context of a problem.	
	9.3	Determine and verify the solution of a system of linear equations graphically, with and without technology.	
	9.4	Explain the meaning of a point of intersection of a system of linear equations.	
	9.5	Determine and verify the solution of a system of linear equations algebraically.	
	9.6	Explain, using examples, why a system of equations may have no solution, one solution or an infinite number of solutions.	
	9.7	Explain a strategy to solve a system of linear equations.	
	9.8	Solve a problem that involves a system of linear equations.	

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics [T] Technology, and Estimation, [V] Visualization

Key Terms

Term	Definition	Example
linear equation		
system of linear equations		
solution to a system		
point of intersection		
infinite solutions		
one solution		
no solutions		
consistent		
inconsistent		
parallel		
perpendicular		

Introduction: Systems of Linear Equations

Challenge

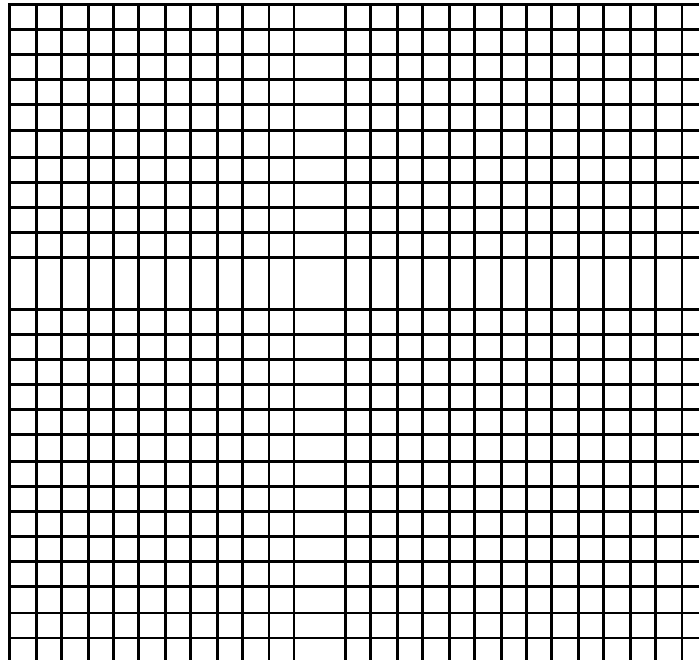
Jazhon is considering two job offers. Concrete Emporium will pay Jazhon a base monthly salary of \$500 plus a commission rate of 5% on all sales each month. All Things Cement offers him a job that pays straight salary, \$2500 per month.

Jazhon wants to consider the two jobs mathematically before he makes his decision. He writes the following equations to represent each job offer.

Concrete Emporium: $E = 0.05s + 500$

All Things Cement: $E = 2500$

1. What does Jazhon need to consider before he can make an educated decision?
2. Graph the two equations on the grid below.

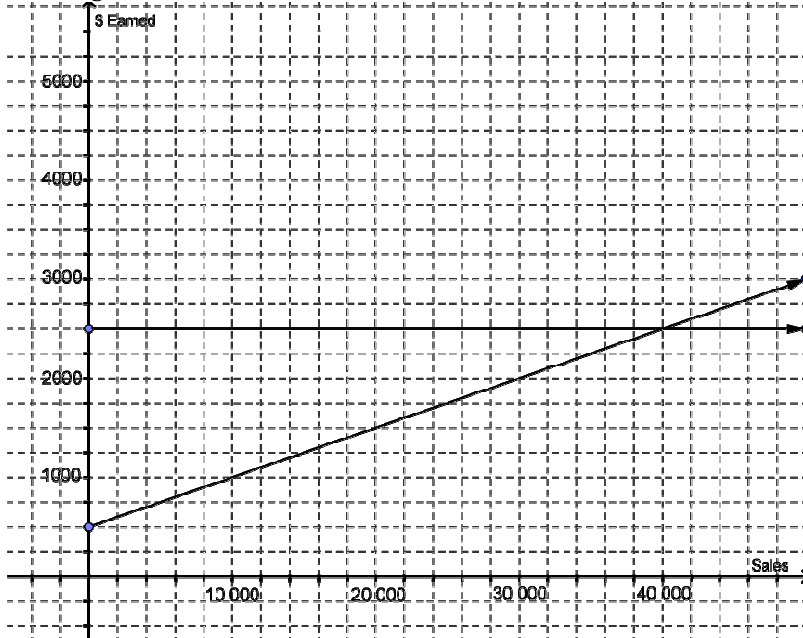


3. What is the significance of the point where the two lines cross?
4. When does the job offered by Concrete Emporium pay more?

Challenge

Concrete Emporium: $E = 0.05s + 500$

All Things Cement: $E = 2500$



We call the scenario to the left a **System of Linear Equations**.

The point (40000, 2500) is on both lines.

We say (40000, 2500) is the **solution to the system**.

That is...it is the point that **satisfies both equations**.

Where the lines cross → earnings are equal.

Concrete Emporium will pay more if Jazhon sells more than \$40 000 worth of concrete.

5. Challenge

Verify that (2,4) is a solution to the following system.

$$\begin{aligned} x + y &= 6 \\ 2x - y &= 0 \end{aligned}$$

Explain your reasoning.

Determine if the given point is a solution to the system of equations. Show your work.

6. Is (1,3) a solution to the following system?
 $y = -2x + 5$
 $y = x + 2$

7. Is (-1,1) a solution to the following system?
 $5x + 6y = 1$
 $6x + 2y = -3$

8. Is (2,1) a solution to the following system?
 $x + 2y = 4$
 $x - y = 1$

9. Is (3,3) a solution to the following system?
 $3y = x + 6$
 $3y = -4x + 21$

10. Is (1,2) a solution to the following system?
 $2x + 2y = 6$
 $y = 4x - 2$

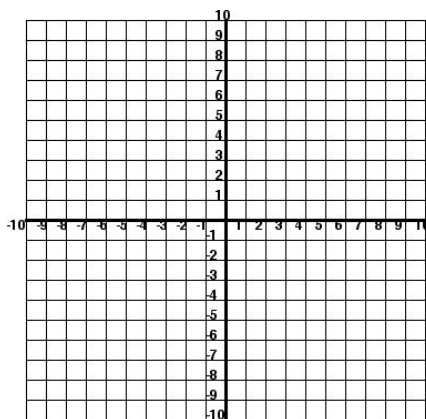
11. Is (-1,1) a solution to the following system?
 $7x = 3y + 10$
 $6x + 5y = -1$

12. Explain how you can determine if a given point is the solution to a system of linear equations.

Challenge

13. Find the solution to the following system of equations.

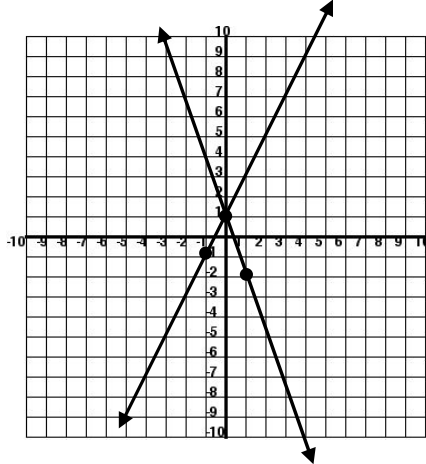
$$y = 2x + 1$$
$$y = -3x + 1$$



Explain your steps and/or thinking.

Find the solution to the following system of equations.

$$y = 2x + 1$$
$$y = -3x + 1$$



Explain your steps and/or thinking.

I graphed each of the lines.

I found the coordinates of the point that is on both lines

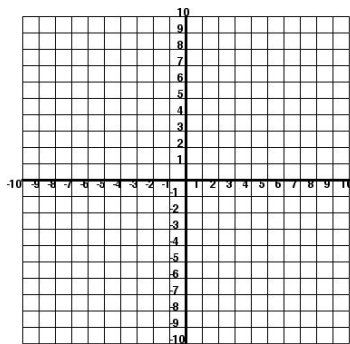
→ where the lines cross!

(0,1)

Solve the following systems by graphing:

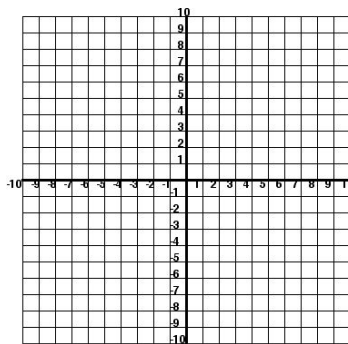
14. Solve:

$$y = 3x - 1$$
$$y = -2x + 4$$



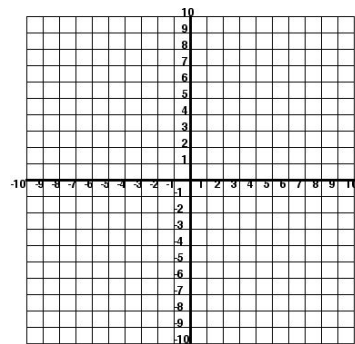
15. Solve:

$$x - y = -2$$
$$4x + 2y = 16$$



16. Solve:

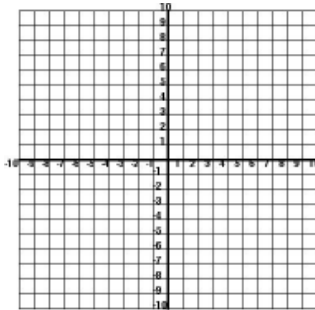
$$x + y = 5$$
$$3x - y = 3$$



Solve the following systems by graphing:

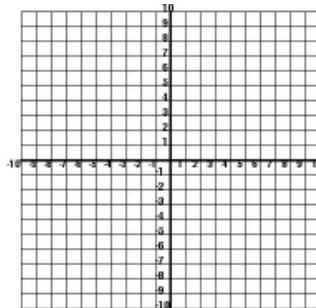
17. Solve:

$x + y = 4$ and $x - y = 2$



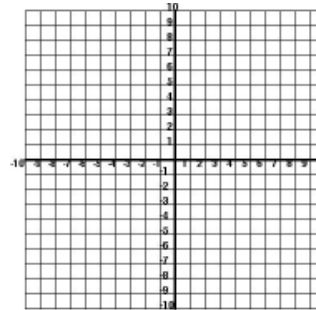
18. Solve:

$y = x - 2$ and $y = \frac{2}{5}x + 1$



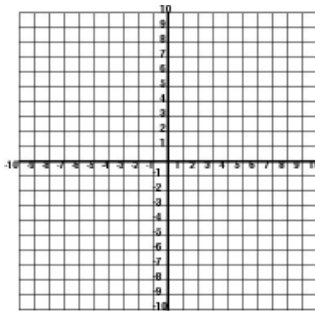
19. Solve:

$y = -3x + 5$ and $x - 2y = 4$



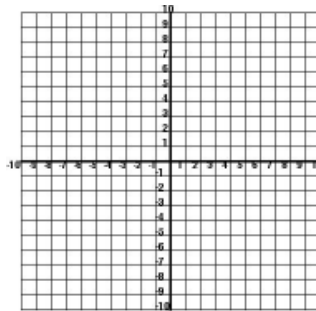
20. Solve:

$x + 2y = 8$ and $3x - y = 3$



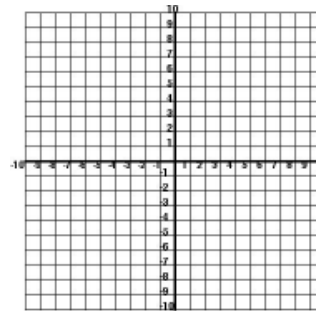
21. Solve:

$5x + 4y = 40$ and $5x + 6y = 60$



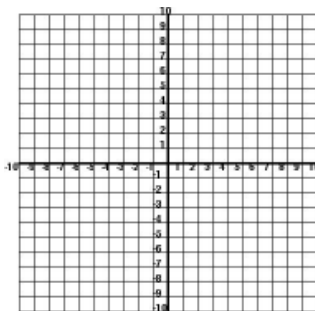
22. Solve:

$x = 5$ and $y + 4 = 10$



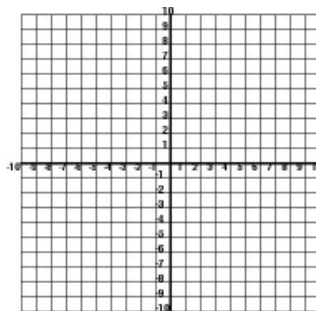
23. Solve:

$y = 2x - 3$ and $y = 2x + 3$



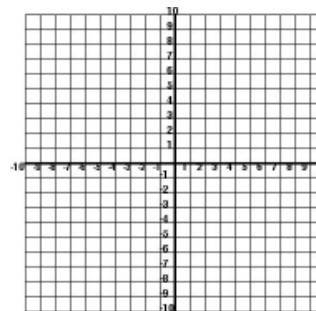
24. Solve:

$x - y = 1$ and $3y = 3x - 3$



25. Solve:

$2y = 3x - 2$ and $4y + 4 = 6x$



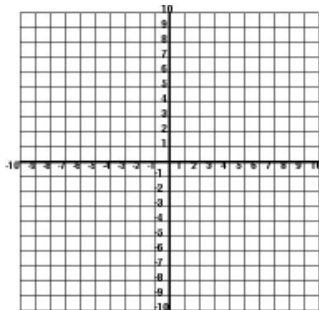
26. What do you notice about the equations above?

27. What do you notice about the equations above?

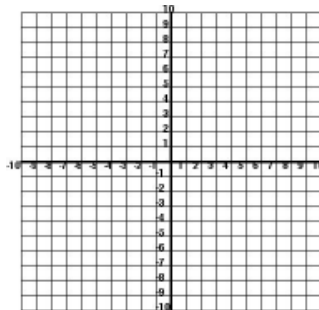
28. What do you notice about the equations above?

29. Challenge

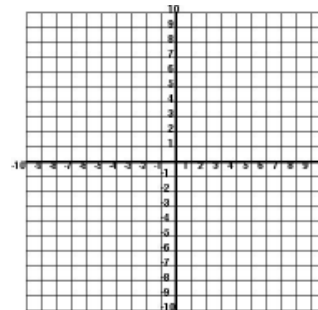
On the three graphs below, draw a system of linear equations with . . .



a) One solution



b) No solutions



c) Infinite Solutions

30. Challenge

How many solutions are there to the system

$$y = 3x + 3$$

$$y = x + 1$$

Explain your reasoning.

Types of Solution Sets:		
<p>One solution</p> <ul style="list-style-type: none"> • Lines intersect once. • Different Slopes. <p>We say the system is CONSISTENT</p>	<p>No Solutions</p> <ul style="list-style-type: none"> • Parallel Lines • Same Slopes • Different y-intercepts <p>We say the system is INCONSISTENT (no solution)</p>	<p>Infinite Solutions</p> <ul style="list-style-type: none"> • Same Lines • Same Slopes • Same y-intercepts <p>We say the system is CONSISTENT</p>

Determine if the following systems have one solution, no solutions, or infinite solutions.

<p>31. $y = 3x + 3$ $y = x + 1$</p> <p>One solution because the slopes are different.</p> <p>Lines will intersect once.</p>	<p>32. $y = 2x + 5$ $y = 3x - 5$</p>	<p>33. $3y = 9x + 12$ $3x - 9y = 12$</p>
<p>34. $6x + 4y = 1$ $3x - 2y = 4$</p>	<p>35. $2x + y = 5$ $y = -2x - 5$</p>	<p>36. $y = \frac{2}{3}x + 5$ $3y = 2x - 5$</p>

Find the value of k that makes each system **inconsistent**.

<p>37. $y = kx - 3$ $2y = 2x + 6$</p>	<p>38. $2y = kx + 1$ $2x - y = 7$</p>	<p>39. $4kx = y - 2$ $5x + 3y - 12 = 0$</p>
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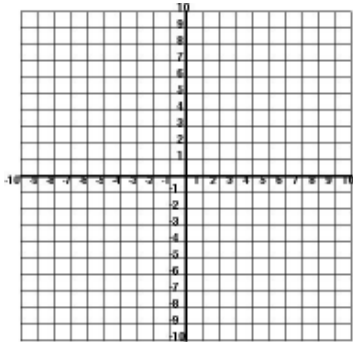
Find the value of b that will produce a system with **infinite solutions**.

<p>40. $y = x - b$ $2y = 2x - 4$</p>	<p>41. $3x - y = 7$ $4y = 12x + b$</p>	<p>42. $2x + 3y - 2b = 0$ $y = -\frac{2}{3}x + 1$</p>
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43. Solve:

$$2x + 3y - 6 = 0$$

$$3x - y + 2 = 0$$



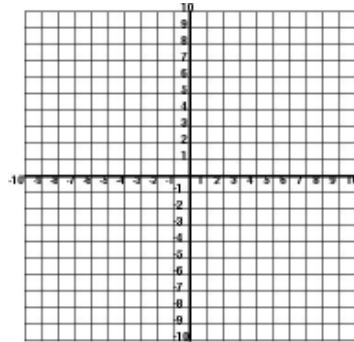
44. The system above is

- a) Consistent
- b) Inconsistent

45. Solve:

$$x - y = 1$$

$$5x + 2y = 5$$



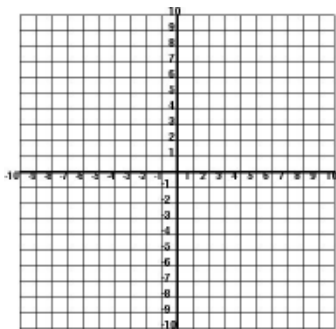
46. Add the two equations above and graph the new equation.

47. What do you notice?

48. Graph the system of equations:

$$y = x + 2$$

$$3y = 2x - 5$$



49. What is the problem when solving this system by graphing?

50. Challenge

Solve the system of linear equations: $y = x + 2$ and $3y = 2x - 5$.

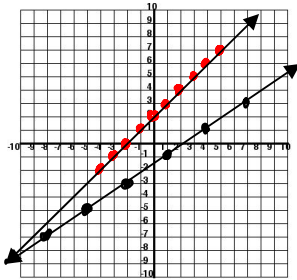
Solving Systems of Equations (without graphing)

Part 1: Solving By substitution.

Graph the system of equations:

$$y = x + 2$$

$$3y = 2x - 5$$



My thoughts...

If I graph each of these lines, I notice that they do not cross at a point that I can easily read on **this** graph.

Also, the second equation is not easily graphed.

I can use a different method.

Algebra! See My Solution Below.

51. What is the solution to a system of linear equations?

52. If a point is present on two lines, what values of that point are equal:

- a. x-values
- b. y-values
- c. both x- and y-values

Solve the system of equations:

"1" $y = x + 2$ I will substitute $(x+2)$ in to equation "2" for y .

"2" $3y = 2x - 5$ $3(x+2) = 2x - 5$

$$3x + 6 = 2x - 5$$

$$x = -11$$

Then substitute $x = -11$ into equation "1".

$$y = (-11) + 2$$

$$y = -9$$

Therefore the solution is $(-11, -9)$

53. Solve the following system of equation without graphing, consider the answers to the previous questions to guide you.

$$y = 2x - 1$$

$$y = -x + 1$$

54. Verify your solution above.

Solve the following systems of equations **by substitution**.

55. Solve.

$$y = 2x - 1$$

$$y = -x + 1$$

Since both $(2x - 1)$ and $(-x + 1)$ are equal to 'y', then they must be equal to each other.

$$2x - 1 = -x + 1$$

$$3x = 2$$

$$x = \frac{2}{3}$$

To find 'y', substitute your known 'x' into either equation.

$$y = -\left(\frac{2}{3}\right) + 1$$
$$y = \frac{1}{3}$$

Solution $\left(\frac{2}{3}, \frac{1}{3}\right)$

56. How can I check the solution to the left?

57. Check the solution to the left.

58. Solve.

$$3x + y = 1$$

$$2x + 3y = 11$$

59. Solve.

$$a + c = 9$$

$$2a + c = 11$$

60. Solve.

$$3x - 4y = -15$$

$$5x + y = -2$$

61. Solve.

$$d + e = 1$$

$$3d - e = 11$$

Solve the following systems of equations **by substitution**.

62. Solve.

$$a + 6b = 9$$

$$3a - 2b = -23$$

63. Solve.

$$2t - w = 13$$

$$4t + 3w = 1$$

64. Solve.

$$3y = -6x + 15$$

$$5y = 5x + 10$$

65. Solve.

$$y = \frac{x}{3} + 2$$

$$3y + 4x = 21$$

66. Solve.

$$3x - 2y = 4$$

$$3x + 4y = 10$$

67. Solve.

$$\frac{1}{4}x + \frac{1}{2}y = 10$$

$$\frac{1}{4}x - \frac{1}{2}y = 0$$

68. Write a system of 2 linear equations for the following problem.

The sum of two numbers is 65. The first number is 17 greater than the second.

69. Find the numbers in the problem to the left.

70. Write a system of 2 linear equations for the following problem.

One number is 12 less than another number. Their sum is 102.

71. Find the numbers in the problem to the left.

72. Write a system of 2 linear equations for the following problem.

Mr. J bought a total of 12 pairs of socks. Athletic socks cost \$5 per pair and dress socks cost \$7 per pair. He spent \$70 in total.

73. How many pairs of each type of socks did he buy?

Part 2: Solving By Elimination (Addition or Subtraction)

Challenge Questions

74. Is (3,1) a solution to the system $2x - y = 5$ and $2x - 4y = 2$?

75. Multiply each of the equations above by 2.

$$2(2x - y = 5) \rightarrow$$

$$2(2x - 4y = 2) \rightarrow$$

76. Is (3,1) still a solution to each of the equations above?

77. Add the two original equations together:

$$2x - y = 5$$

$$\underline{2x - 4y = 2}$$

78. Is (3,1) a solution to the new equation?

79. What conclusions can you draw about adding/subtracting equations together?

80. What conclusions can you draw about multiplying equations in a system by a constant?

81. Can you multiply the equations by different numbers without affecting the solution?

82. Graph equation ①:

① $2x + y = 8$

83. Graph equation ②:

② $y = 4x - 4$

84. Add equations ① and ②.

Call this equation ③.

③ _____

85. Graph equation ③.

86. Multiply ③ $\times 3$ and call this equation ④.

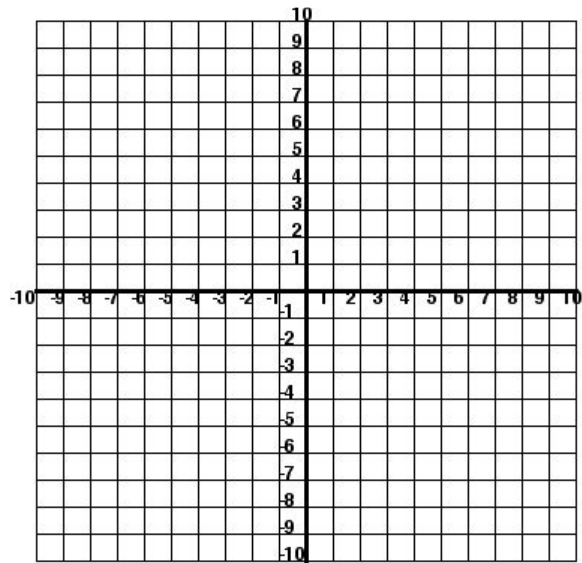
④ _____

87. Graph equation ④.

88. Add ③ and ④, call this equation ⑤.

⑤ _____

89. Graph equation ⑤.



90. Describe what you see happening above.

91. Write a set of rules describing what you may do to a system of equations in order to find the solution. That is, how can you manipulate the equations without affecting the solution?

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92. Add the two equations together, then solve.

$$3x - 6y = 21$$

$$\underline{-3x - 4y = -1}$$

$$-10y = 20$$

$$y = -2 \quad \rightarrow \quad 3x - 6(-2) = 21$$

$$3x + 12 = 21$$

$$3x = 9$$

$$x = 3$$

Solution: (3, -2)

93. Solve.

$$2x + 3y = 18$$

$$2x - 3y = -6$$

94. Solve.

$$8x + 2y = -20$$

$$2x - 2y = -30$$

95. Solve.

$$-4t + 3s = 2$$

$$8t - 6s = -4$$

96. Solve.

$$6x - 3y = 24$$

$$x + y = -2$$

97. Solve.

$$3b - a = 1$$

$$-12b + 4a = -4$$

98. Solve.

$$0.05x + 0.07y = 19$$

$$x + y = 300$$

99. Solve.

$$x + y = 1200$$

$$0.20x + 0.40y = 36$$

100. Two numbers have a sum of 25 and a difference of 7. What are the two numbers?

101. Anya has a pocket full of loonies (\$1 coins) and toonies (\$2 coins). She has \$41 in total. If she has 29 coins, how many of each does she have?

102. When three times one number is added to two times another number, the sum is 21. When 4 times the second number is subtracted from 10 times the first number, the difference is 38. What are the numbers?

103. The total cost (before taxes) for three coffees and two cookies is \$10.05. The cost for five coffees and three cookies is \$16.10. Find the individual cost for each item.

Solving Problems with Systems of Equations. Use the method of your choice.

104. A job offered to Mr. Xu will pay straight commission at a rate of 6% on all sales. A second job offer will pay a monthly salary of \$400 and 2% commission. How much would Mr. Xu have to sell so that both jobs would pay him the same amount.

105. In his 2004-05 season, Steve Nash scored 524 total baskets (not including free throws). He scored 336 more two point baskets than three point baskets. Write and solve a system of linear equations that represents this problem.

When would the job paying straight commission be a better choice?

Interpret your solution:

106. Mr. J has a class with 30 students in it. 22 of those students own a cell phone. $\frac{4}{5}$ of the girls owned a cell phone and $\frac{3}{5}$ of the boys owned a cell phone. How many girls were in this class?

107. Daiki invested a total of \$12 000 in two stocks in 2009. One stock earned 4% interest and the other earned 7% interest. Daiki earned a total of \$615 in interest in 2009. How much did he invest in each stock?

For each of the following problems, write and solve a system of equations. Interpret solutions!

108. Breakers Volleyball sold 570 tickets to their home opener, some tickets cost \$2 and some cost \$5. The total revenue was \$1950. How many of each type of ticket were sold?

109. Mr. J is doing routine maintenance on his old farm truck. This month he spent \$26.50 on 6 litres of oil and 2 gaskets. Last month he spent \$25.00 on 4 litres of oil and 4 gaskets. Find the price of each gasket and one litre of oil.

110. Anya makes a trip to the local grocery store to buy some bulk candy. She chooses two of her favourite candies, gummy frogs and gummy penguins. Gummy frogs sell for \$1.10 per 100g and penguins sell for \$1.75 per 100g. Anya buys a total of 500g of candy for \$7.84 (no taxes). How much of each type did she buy?

111. For his Christmas party, Teems Prey is making a bowl of exotic punch for the kid's table. Imported lychee juice sells for \$12.50 per litre and guava nectar sells for \$18 per litre. He is making 8 litres and will need to pay \$126.40 for the perfect blend. How much of each type does he use?

112. Jay Maholl swam 12 km downstream in Englishman River in two hours. The return trip upstream took 6 hours. Find the speed of the current in Englishman River.

114. The Lucky-Lady dinghy travels 25 km upstream in five hours. The return trip takes only half an hour. Find the speed of the boat and the speed of the current.

113. (What assumption must you make?)

115. A bumble bee travels 4.5 km into a headwind in 45 minutes. The return trip with the wind only takes 15 minutes. Assuming speeds are constant, find the speed of the bumble bee in still air.

116. A plane flew a distance of 650 km in 3.25 hours when travelling in a tailwind. The return trip took 6.5 hours against the same wind. Assume both speeds are constant. Find the speed of the plane and the wind speed.

117. A 50% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 50% acid solution. How much of each stock solution should she use?

Let x = volume of 20% solution
Let y = volume of 70% solution.

$$x + y = 20$$
$$0.2x + 0.7y = (0.5)(20)$$

Solve the System:

118. A 65% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 65% acid solution. How much of each stock solution should she use?

119. The karat (or carat) is a measure of the purity of gold in gold alloy. 18K gold is approximately 75% pure and 14K gold is approximately 58.5% pure. Using 18K and 14K stock, a goldsmith needs to produce 40g of gold alloy that is 70% pure. How much of each stock will he need to use? (round to nearest hundredth)

120. A goldsmith needs to make 50g of 14K gold (58.5%) from 18K (75%) and 10K (41.7%) stock alloys. How much of each does she need? (round to nearest hundredth)