HW Mark: 10 9 8 7 6 RE-Submit

The Real Number System

This booklet belongs to: _______________Period___

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
		Pg.	
		Pg.	
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		Pg.	
		REVIEW	
		TEST	

Your teacher has important instructions for you to write down below.

The Real Number System

STRAND Algebra & Number		DAILY TOPIC	EXAMPLE
B1. Demonstrate an understanding of factors of whole numbers by determining the: • Prime factors • Greatest Common Factor • Least Common Multiple • Square root • Cube root	1.1	Determine the prime factors of a whole number.	
	1.2	Explain why the numbers 0 and 1 have no prime factors.	
	1.3	Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process	
	1.4	Determine, concretely, whether a given whole number is a perfect square, a perfect cube or neither.	
	1.5	Determine, using a variety of strategies, the square root of a perfect square, and explain the process.	
	1.6	Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.	
	1.7	Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots.	
B2. Demonstrate an understanding of irrational numbers by: • Representing, identifying and simplifying irrational numbers. • Ordering irrational numbers	2.1	Sort a set of numbers into rational and irrational numbers.	
	2.2	Determine an approximate value of a given irrational number.	
	2.3	Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning.	
	2.4	Order a set of irrational numbers on a number line.	
	2.5	Express a radical as a mixed radical in simplest form (limited to numerical radicands).	
	2.6	Express a mixed radical as an entire radical (limited to numerical radicands).	
	2.7	Explain, using examples, the meaning of the index of a radical.	
	2.8	Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).	

[C] Communication **[PS]** Problem Solving, **[CN]** Connections **[R]** Reasoning, **[ME]** Mental Mathematics **[T]** Technology, and Estimation, **[V]** Visualization

Key Terms

Term	Definition	Example
Real Number (R)		
Rational Number (Q)		
Irrational Number ($\bar{Q}$)		
Integer (Z)		
Whole Number (W)		
Natural Number (N)		
Factor		
Factor Tree		
Prime Number		
Prime Factorization		
GCF		
Multiple		
LCM		
Radical		
Index		
Root Square root Cube root		
Power		
Entire Radical		
Mixed Radical		

The Real Number System

Real numbers are the set of numbers that we can place on the number line.

Real numbers may be positive, negative, decimals that repeat, decimals that stop, decimals that don't repeat or stop, fractions, square roots, cube roots, other roots. Most numbers you encounter in high school math will be real numbers.

The square root of a negative number is an example of a number that does not belong to the Real Numbers.

There are 5 subsets we will consider.

Real Numbers

Rational Numbers (Q)

Numbers that can be written in the form $\frac{m}{n}$ where m and n are both integers and n is not 0.

Rational numbers will be terminating or repeating decimals.

Eg. 5, -2. 3, $\frac{4}{3}$, $2\frac{3}{8}$

Natural (N)	Whole (W)	Integers (Z)
{1, 2, 3,...}	{0, 1, 2, 3,...}	{...,-3,-2,-1, 0, 1, 2, 3,...}

Irrational Numbers ($\bar{Q}$)

Cannot be written as $\frac{m}{n}$.
Decimals will not repeat, will not terminate.

Eg. $\sqrt{3}$, $\sqrt{7}$, π, 53.123423656787659...

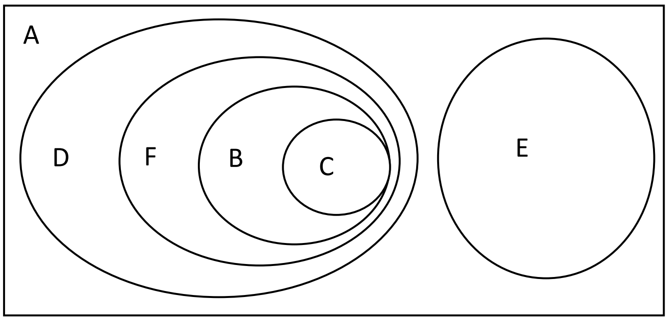
Name all of the sets to which each of the following belong?

1. 8	2. $\frac{4}{5}$	3. $\frac{15}{5}$
4. $\sqrt{7}$	5. $\sqrt{0.5}$	6. $12.3\overline{4}$
7. -17	8. $-(\frac{2}{3})^3$	9. $2.7328769564923\ldots$

Write each of the following Real Numbers in decimal form. Round to the nearest thousandth if necessary. Label each as Rational or Irrational.

10. $\frac{2}{9}$	11. $-3\frac{3}{7}$	12. $\sqrt{8}$
13. $\sqrt[3]{9}$	14. $\sqrt[4]{256}$	15. $\sqrt[5]{25}$

16. Fill in the following diagram illustrating the relationship among the subsets of the real number system. (Use descriptions on previous page)



A____________________________

B____________________________

C____________________________

D____________________________

E____________________________

F____________________________

17. Place the following numbers into the appropriate set, rational or irrational.

$5, \quad \sqrt{2}, \quad 2.\overline{13}, \quad \sqrt{16}, \quad \frac{1}{2}, \quad 5.1367845\ldots, \quad \frac{\sqrt{7}}{2}, \quad \sqrt[3]{8}, \quad , \quad \sqrt[3]{25}$

Rational Numbers

Irrational Numbers

18. Which of the following is a rational number?

a. $\frac{\sqrt{3}}{2}$
b. $\sqrt[3]{16}$
c. $\frac{5}{7}$
d. $12.356528349875\ldots$

19. Which of the following is an irrational number?

a. $\sqrt{\frac{16}{9}}$
b. π
c. $\frac{3}{8}$
d. $\sqrt[3]{27}$

20. To what sets of numbers does -4 belong?

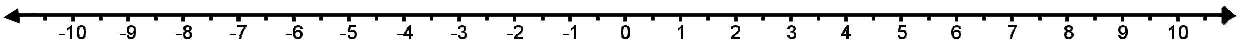
a. natural and whole
b. irrational and real
c. integer and whole
d. rational and integer

21. To what sets of numbers does $-\frac{4}{3}$ belong?

a. natural and whole
b. irrational and real
c. integer and whole
d. rational and real

Your notes here…

The Real Number Line



All real numbers can be placed on the number line. We could never list them all, but they all have a place.

Estimation:

It is important to be able to estimate the value of an irrational number. It is one tool that allows us to check the validity of our answers.

Without using a calculator, estimate the value of each of the following irrational numbers.
Show your steps!

22. $\sqrt{7}$

Find the perfect squares on either side of 7.
→4 and 9
Square root 4 = 2
Square root 9 = 3

Guess & Check:
2.6 x 2.6 =6.76
2.7 x 2.7 = 7.29
$\therefore \sqrt{7}$ is about 2.6

23. $\sqrt{14}$

24. $\sqrt{75}$

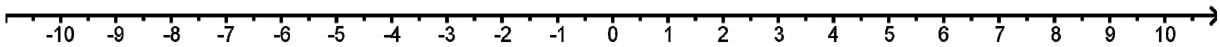
25. $\sqrt[3]{11}$

26. $\sqrt[3]{90}$

27. $\sqrt[3]{150}$

28. Place the corresponding letter of the following Real Numbers on the number line below.

A. -6 B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. $5\frac{1}{4}$ E. $\sqrt{2}$ F. $-\sqrt{7}$ G. $\frac{\sqrt{3}}{2}$ H. $-\frac{\sqrt{4}}{3}$



Factors, Factoring, and the Greatest Common Factor

We often need to find factors and multiples of integers and whole numbers to perform other operations.

For example, we will need to find common multiples to add or subtract fractions.
For example, we will need to find common factors to reduce fractions.

Factor: (NOUN)

Factors of 20 are {1,2,4,5,10,20} because 20 can be evenly divided by each of these numbers.
Factors of 36 are {1,2,3,4,6,9,12,18,36}
Factors of 198 are { 1,2,3,6,9,11,18,22,33,66,99,198}

Use division to find factors of a number. Guess and check is a valuable strategy for numbers you are unsure of.

To Factor: (VERB) The act of writing a number (or an expression) as a product.

To factor the number 20 we could write 2×10 or 4×5 or 1×20 or $2 \times 2 \times 5$ or $2^2 \times 5$.
When asked to factor a number it is most commonly accepted to write as a product of prime factors.
Use powers where appropriate.

Eg. $20 = 2^2 \times 5$ Eg. $36 = 2^2 \times 3^2$ Eg. $198 = 2 \times 3^2 \times 11$

Prime:
When a number is only divisible by 1 and itself, it is considered a prime number.

A factor tree can help you "factor" a number.

```
          36
        /    \
     4          9
    / \        / \
   2   2      3   3
```

$\therefore\ 36 = 2^2 \times 3^2$

Write each of the following numbers as a product of their prime factors.

29. 100	30. 120	31. 250

Write each of the following numbers as a product of their prime factors.

32. 324	33. 1200	34. 800

Greatest Common Factor

At times it is important to find the largest number that divides evenly into two or more numbers…the **Greatest Common Factor (GCF)**.

Challenge:

35. Find the GCF of 36 and 198.

Challenge:

36. Find the GCF of 80, 96 and 160.

Some Notes…

Find the GCF of each set of numbers.

37. 36, 198

$36 = 2^2 \times 3^2$
$198 = 2 \times 3^2 \times 11$

Prime factors in common are 2 and 3^2.

GCF is $2 \times 3^2 = 18$

(Alternate method: List all factors…choose largest in both lists.)

38. 98, 28

39. 80, 96, 160

$80 = 2^4 \times 5$
$96 = 2^5 \times 3$
$160 = 2^5 \times 5$

Prime factors in common are 2^4.

GCF is $2^4 = 16$

(Alternate method: List all factors…choose largest in both lists.)

40. 24, 108

41. 126, 189, 735, 1470

42. 504, 1050, 1386

Multiples and Least Common Multiple

Challenge

43. Find the first seven multiples of 8.

Challenge

44. Find the least common multiple of 8 and 28.

Multiples of a number

Multiples of a number are found by multiplying that number by {1,2,3,4,5,…}.

Find the first five multiples of each of the following numbers.

45. 8

46. 28

47. 12

Find the least common multiple of each of the following sets of numbers.

48. 8,28

$8 = 2^3$
$28 = 2^2 \times 7$

Look for largest power of each prime factor…

In this case, 2^3 and 7.
$LCM = 2^3 \times 7$
$LCM = 56$

49. 72,90

50. 25, 220

51. 8, 12, 22

52. 4, 15, 25

53. 18, 20, 24, 36

54. Use the least common multiple of 2, 6, and 8 to add:
$$\frac{3}{8}+\frac{5}{6}+\frac{1}{2}$$

55. Use the least common multiple of 2, 5, and 7 to evaluate:
$$\frac{3}{5}-\frac{2}{7}+\frac{3}{2}$$

56. Use the least common multiple of 3, 8, and 9 to evaluate:
$$\frac{7}{9}-\frac{1}{3}-\frac{1}{8}$$

Radicals:

Radicals are the name given to square roots, cube roots, quartic roots, etc.

$$\sqrt[n]{x}$$

The parts of a radical:

Radical sign	$\sqrt{\ }$	(Operations under the radical are evaluated as if inside brackets.)
Index	n	(tells us what type of root we are looking for, if blank...index is 2)
Radicand	x	(the number to be "rooted")

A radical and its simplified form are equivalent expressions.

Square Roots

Square root of 81 looks like $\sqrt{81}$. It means to find what value must be multiplied by itself twice to obtain the number we began with.

$\sqrt{81}$ *we think* ... $81 = 9 \times 9 \rightarrow \sqrt{81} = 9$ $\sqrt{a^4}$ we think ...$a^4 = a^2 \times a^2 \rightarrow \sqrt{a^4} = a^2$

PERFECT SQUARE NUMBER: A number that can be written as a product of two equal factors.

$81 = 9 \times 9$ } 81 is a perfect square. Its square root is 9.

First 15 Perfect Square Numbers:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

Your notes here…

Operations inside a $\sqrt{}$ must be considered as if they were inside brackets...do them first!

Evaluate the following.

57. $\sqrt{49}$

58. $\sqrt{-25}$

59. $-\sqrt{36}$

60. Finish the statement:

I know that $\sqrt{16} = 4$ because...

61. Finish the statement:

I know that $\sqrt{\frac{64}{81}} = \frac{8}{9}$ because...

62. Finish the statement:

I know that $\sqrt{-36} \neq -6$ because...

63. $\sqrt{121}$

64. $\sqrt{45-20}$

65. $2\sqrt{40-(-9)}$

66. Simplify. $\sqrt{x^2}$

67. Simplify. $\sqrt{4x^2}$

68. Simplify. $\sqrt{16x^4}$

Cube Roots:

PERFECT CUBE NUMBER: A number that can be written as a product of three equal factors.

Cube root of 64 looks like $\sqrt[3]{64}$.
The index is 3. So we need to multiply our answer by itself 3 times to obtain 64. $4 \times 4 \times 4 = 64$

First 10 Perfect Cube Numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

Evaluate or simplify the following.

69. $\sqrt[3]{8}$
Explain what the small 3 in this problem means.

70. $\sqrt[3]{8}$

71. How could a factor tree be used to help find $\sqrt[3]{125}$?

72. Evaluate $\sqrt[3]{125}$.

73. $\sqrt[3]{-27}$

74. $\sqrt[3]{1000}$

75. $\sqrt[3]{-8}$

76. Show how prime factorization can be used to evaluate $\sqrt[3]{27}$.

77. $\sqrt[3]{343}$

78. $\sqrt[3]{-216}$

79. $\sqrt[3]{27} \times \sqrt{20 \times 5}$

80. $\sqrt[3]{64} \times \sqrt{45 - 20}$

81. $\sqrt[3]{-125}$

82. $\sqrt[4]{a^{12}}$

83. $\sqrt[3]{a^6}$

84. $\sqrt[3]{8x^3}$

Other Roots.

85. How does $\sqrt[6]{729}$ differ from $\sqrt[3]{729}$? Explain, do not simply evaluate.

86. Evaluate if possible. $\sqrt[4]{16}$

87. Evaluate if possible. $\sqrt[4]{-16}$.

88. Evaluate if possible. $\sqrt[5]{32}$.

89. Evaluate if possible. $\sqrt[4]{81}$.

90. Evaluate if possible. $\sqrt[6]{64}$.

91. Evaluate if possible. $\sqrt[3]{24-16}$.

92. Evaluate if possible. $\sqrt[4]{2(32-24)}$.

93. Evaluate if possible. $\sqrt[3]{4(5-3)}$.

Using a calculator, evaluate the following to two decimal places.

94. $\sqrt[3]{27}-\sqrt[5]{27}$

95. $2\sqrt{10}+\sqrt[4]{64}$

96. $\sqrt[5]{-32}-\sqrt[4]{16}$

97. $19-\sqrt[3]{18}$

98. $\frac{\sqrt{12}-\sqrt[3]{7}}{2}$

99. $\frac{\sqrt[3]{9}-\sqrt[3]{27}}{3}$

100. Describe the difference between radicals that are rational numbers and those that are irrational numbers.

Evaluate or simplify the following.

101. $\sqrt[3]{125}$	102. $\sqrt{2(15-(-3))}$	103. $\sqrt{\sqrt{16}}$
104. $\sqrt{0.16}$	105. $\sqrt{0.0001}$	106. $3\sqrt{25}-4\sqrt[3]{8}$
107. $\sqrt{\frac{1}{4}}$	108. $\sqrt{\frac{16}{49}}$	109. $\sqrt{\frac{100}{400}}$
110. $\sqrt{a^4}$	111. $\sqrt[3]{-x^6}$	112. $\sqrt[3]{8x^3}$

Evaluate or simplify the following.

113. $\sqrt{5^2}$

114. $\left(\sqrt{5}\right)^2$

115. $-\sqrt{(-5)^2}$

116. $\left(\sqrt{49}-\sqrt{64}\right)^3$

117. $\sqrt{\sqrt{16}+\sqrt{25}}$

118. What would be the side length of a square with an area of 1.44 cm²?

119. $\left(\sqrt[4]{16}\right)^3$

120. $\sqrt[5]{-32}$

121. $\sqrt[8]{256}$

122. Use the prime factors of 324 to determine if 324 is a perfect square. If so, find $\sqrt{324}$.

Answer:
$324 = 2^2 \times 3^4$ if fully factored
$\therefore \sqrt{324} = \sqrt{2 \times 2 \times 3^2 \times 3^2}$
$\therefore \sqrt{324} = \sqrt{(2 \times 3^2) \times (2 \times 3^2)}$
$\therefore \sqrt{324} = (2 \times 3^2)$
$\therefore \sqrt{324} = 18$

123. Use the prime factors of 576 to determine if 576 is a perfect square. If so, find $\sqrt{576}$.

124. Use the prime factors of 1728 to determine if it is a perfect cube. If so, find $\sqrt[3]{1728}$.

125. Use the prime factors of 5832 to determine if it is a perfect cube. If so, find $\sqrt[3]{5832}$.

126. An engineering student developed a formula to represent the maximum load, in tons, that a bridge could hold. The student used 1.7 as an approximation for $\sqrt{3}$ in the formula for his calculations. When the bridge was built and tested in a computer simulation, it collapsed. The student had predicted the bridge would hold almost three times as much.

The formula was:
$5000(140-80\sqrt{3})$

What weight did the student think the bridge would hold?

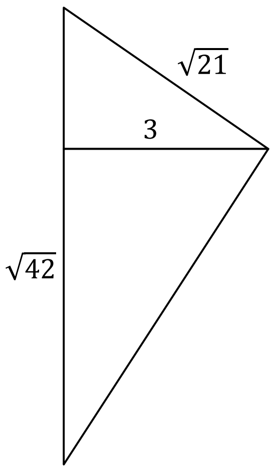
Calculate the weight the bridge would hold if he used $\sqrt{3}$ in his calculator instead.

127. For what values of x is $\sqrt{x-2}$ <u>not defined</u>?

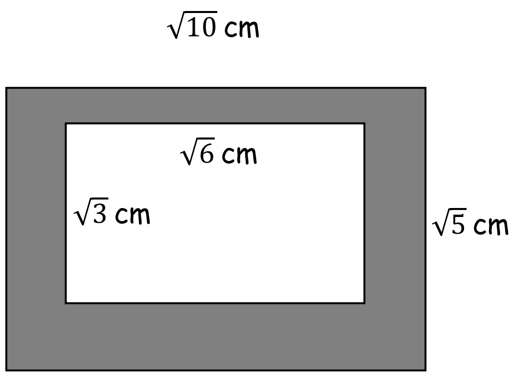
128. For what values of x is $\sqrt{x+3}$ <u>not defined</u>

129. For what values of x is $\sqrt{5-x}$ <u>not defined</u>

130. Calculate the perimeter to the nearest tenth. The two smaller triangles are <u>right</u> triangles.



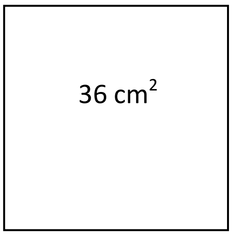
Calculate the area of the shaded region.



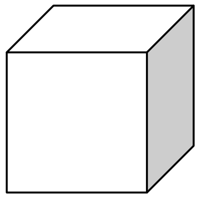
131. To the nearest tenth:

132. As an equation using radicals:
(you may need to come back to this one)

133. Consider the square below. Why might you think $\sqrt{\ }$ is called a square root?



134. Consider the diagram below. Why do you think $\sqrt[3]{\ }$ is called a cube root?



64 cm^3

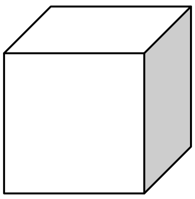
135. Find the side length of the square above.

136. Find the edge length of the cube above.

137. Why do you think 81 is called a "perfect square" number?

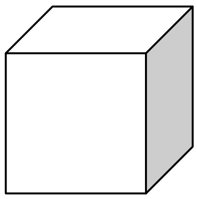
138. Why do you think 729 is called a "perfect cube" number?

139. Find the surface area of the following cube.



125 cm^3

140. Find the surface area of the following cube.



216 cm^3

141. A cube has a surface area of 294 m^2. Find its edge length in centimetres.

142. A cube has a surface area of 1093.5 m^2. Find its edge length in centimetres.

Multiplying Radicals.

Some notes possibly…

143. Challenge

Evaluate $\sqrt{4} \times \sqrt{9}$

144. Challenge

What single radical has the same value as $\sqrt{4} \times \sqrt{9}$?

What is the product of the radicands?

145. Challenge

Evaluate $\sqrt{16} \times \sqrt{4}$

146. Challenge

What single radical has the same value as $\sqrt{16} \times \sqrt{4}$?

What is the product of the radicands?

147. Based on the examples above, can you write a rule for multiplying radicals?

148. Challenge

Evaluate: $2\sqrt{9} \times 5\sqrt{4}$

Multiplying Radicals: The Multiplication property

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$ (this is reversible)

Evaluate	Notice…	Rule:
$\sqrt{4} \times \sqrt{9}$ 2×3 $= 6$	$\sqrt{4} \times \sqrt{9}$ $\sqrt{4 \times 9}$ $= \sqrt{36}$ $= 6$	$\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9} = \sqrt{36} = 6$

Evaluate	Notice…	Rule:
$\sqrt{16} \times \sqrt{4}$ 4×2 $= 8$	$\sqrt{16} \times \sqrt{4}$ $\sqrt{16 \times 4}$ $= \sqrt{64}$ $= 8$	$\sqrt{16} \times \sqrt{4} = \sqrt{16 \times 4} = \sqrt{64} = 8$

Evaluate	Notice…
$2\sqrt{9} \times 5\sqrt{4}$ $2 \times 3 \times 5 \times 2$ $= 60$	$2\sqrt{9} \times 5\sqrt{4}$ $= 2 \times 5 \times \sqrt{9 \times 4}$ $= 10\sqrt{36}$ $= 60$

Your notes here…

Multiply each of the following. Leave answers in radical form if necessary. We will simplify radicals fully in a later section.

149. $\sqrt{6} \times \sqrt{2}$

150. $\sqrt{8} \times \sqrt{2}$

151. $\sqrt{7} \times \sqrt{3}$

152. $-\sqrt{7} \times \sqrt{7}$

153. $\sqrt{3} \times -\sqrt{3}$

154. $3\sqrt{18} \times -2\sqrt{12}$

155. $-\sqrt{5} \times 2\sqrt{20}$

156. $-10\sqrt{3} \times -\frac{\sqrt{5}}{5}$

157. $\left(\frac{3}{4}\sqrt{2}\right)\left(-\frac{2}{3}\sqrt{3}\right)$

158. $\left(\frac{3}{4}\sqrt{6}\right)\left(-\frac{2}{3}\sqrt{6}\right)$

159. $\frac{\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{5}}{\sqrt{2}}$

160. $\frac{2\sqrt{5}}{\sqrt{3}} \times \frac{5\sqrt{2}}{15\sqrt{7}}$

161. Challenge

Write $\sqrt{50}$ as a product of two radicals as many ways as you can (whole number radicands only).

Find the pair from above that includes the largest perfect square and write it here→

Simplify the perfect square in that pair→

162. Challenge

Simplify $2\sqrt{20}$ using the previous example.
(Think of it as $2 \times \sqrt{20}$.)

Explain your process...

163. What is a mixed radical?

164. Challenge
Evaluate:
$2\sqrt{3} \times \sqrt{6}$

Explain in your words:

165. Challenge
Evaluate:
$(-3\sqrt{6})(5\sqrt{8})$

Explain in your words:

Your notes here...

Radicals as equivalent expressions:

Eg. 2 and $\frac{6}{3}$ are equivalent expressions. They occupy the same place on the number line.
As do $\sqrt{12}$ and $2\sqrt{3}$.

Simplifying radicals gives us a standard way to express numbers. We will follow particular patterns so that each of us writes our answers in the same form. Working in radical form allows us to round answers at the end of our calculations if necessary, creating more accurate solutions.

Simplifying Radicals:

Like fractions, radicals must be simplified to "lowest terms". To do this we must consider what type of radical we are working with.

We will remove part of the number under the radical sign **IF** an appropriate factor can be found.

To simplify square roots, we look for perfect square factors. We then remove the perfect square from under the radical sign.

Simplify. $\sqrt{50}$

$\sqrt{50}$ is called an **entire** radical.
This is not a perfect square, but 50 has a perfect square factor, 25.

$\sqrt{50} = \sqrt{25 \times 2}$ We know the square root of 25…it is 5. We cannot simplify $\sqrt{2}$.
$\sqrt{25 \times 2} = 5 \times \sqrt{2}$ We write this as a **mixed radical.**

$= 5\sqrt{2}$

Simplify. $2\sqrt{20}$

$2\sqrt{20} = \ 2 \times \sqrt{20}$ This reads "2 times the square root of 20."
$2 \times \sqrt{20} = \ 2 \times \sqrt{4 \times 5}$ We must now simplify $\sqrt{20}$. 20 has a perfect square factor, 4.
$2 \times \sqrt{4 \times 5} = 2 \times 2 \times \sqrt{5}$ We write this as a **mixed radical**.

$= 4\sqrt{5}$

Multiply. Answer as a mixed radical.

$2\sqrt{3} \times \sqrt{6}$
$2\sqrt{3 \times 6}$ We can multiply non-radical numbers and we can multiply radicands.
$= 2\sqrt{18}$ Now simplify the new radical.
$= 2 \times \sqrt{9 \times 2}$ The radicand, 18, has a perfect square factor, 9.
$= 2 \times 3 \times \sqrt{2}$ Write as a **mixed radical.**

$= 6\sqrt{2}$

Key process:

Entire radical

vs

Mixed radical

Multiply. Answer as a mixed radical.

$(-3\sqrt{6})(5\sqrt{8})$
$= (-3 \times 5 \times \sqrt{6} \times \sqrt{8})$ Multiply non-radicals, multiply radicands
$= -15 \times \sqrt{48}$
$= -15\sqrt{48}$
$= -15 \times \sqrt{16 \times 3}$ Simplify radical
$= -15 \times 4 \times \sqrt{3}$
$= -60\sqrt{3}$

Alternative method: Factorization of Radicand

To simplify square roots, we can write the **radicand** as a product of its primes.
We then look for factors that are present twice (square roots) or three times (cube roots).
We then remove the perfect square from under the radical sign.

Simplify. $\sqrt{50}$

$\sqrt{50} = \sqrt{5 \times 5 \times 2}$ When a factor is present twice, it can be removed (as a single) from under the radical.

$= 5 \times \sqrt{2}$ We write this as a **mixed radical**.

$= 5\sqrt{2}$

Simplify. $3\sqrt{20}$

$3\sqrt{20} = \ 3 \times \sqrt{20}$

$3 \times \sqrt{20} = \ 3 \times \sqrt{(\mathbf{2} \times \mathbf{2}) \times 5}$ The factor 2 is present twice, it comes out as 2.

$3 \times \sqrt{4 \times 5} \ = 3 \times \mathbf{2} \times \sqrt{5}$ Multiply the two rational numbers in front the radical.

$= 6\sqrt{5}$

Multiply $2\sqrt{3} \times \sqrt{6}$. Answer as a mixed radical.

$2\sqrt{3} \times \sqrt{6}$

$2 \times \sqrt{3} \times \sqrt{3 \times 2}$ We can multiply radicands.

$= 2\sqrt{(\mathbf{3} \times \mathbf{3}) \times 2}$ Now simplify the new radical.

$= 2 \times \mathbf{3} \times \sqrt{2}$

$= 2 \times 3 \times \sqrt{2}$ Write as a **mixed radical.**

$= 6\sqrt{2}$

Multiply. Answer as a mixed radical.

$(-3\sqrt{6})(5\sqrt{8})$

$= (-3 \times 5 \times \sqrt{6} \times \sqrt{8})$ Multiply non-radicals, multiply radicands

$= (-3 \times 5 \times \sqrt{2 \times 3} \times \sqrt{2 \times 2 \times 2})$

$= -15 \times \sqrt{(2 \times 2) \times (2 \times 2) \times 3}$ Notice there are two pairs of like factors

$= -15 \times 2 \times 2 \times \sqrt{3}$

$= -60\sqrt{3}$

Express each of the following as mixed radicals in simplest form.

166. $\sqrt{8}$	167. $\sqrt{75}$	168. $\sqrt{48}$
169. $\sqrt{12}$	170. $\sqrt{200}$	171. $\sqrt{128}$
172. $-\sqrt{240}$	173. $\sqrt{1200}$	174. $\sqrt{7200}$

Simplify the following.

175. $2\sqrt{27}$	176. $-3\sqrt{32}$	177. $5\sqrt{25}$
178. $-4\sqrt{12}$	179. $3\sqrt{50}$	180. $-4\sqrt{20}$
181. $\frac{1}{2}\sqrt{24}$	182. $\frac{3}{4}\sqrt{108}$	183. $-\frac{4}{3}\sqrt{27}$

Simplify the following.

184. $0.25\sqrt{8}$	185. $-1.5\sqrt{80}$	186. $-2.4\sqrt{48}$

Simplify the following cube roots.

187. $\sqrt[3]{16}$	188. $\sqrt[3]{54}$	189. $\sqrt[3]{2000}$
190. $-\sqrt[3]{56}$	191. $\sqrt[3]{432}$	192. $\sqrt[3]{1458}$

Simplify the following cube roots.

193. $3\sqrt[3]{81}$	194. $-2\sqrt[3]{32}$	195. $-6\sqrt[3]{24}$
196. $\frac{1}{3}\sqrt[3]{54}$	197. $-\frac{2}{5}\sqrt[3]{5000}$	198. $\frac{3}{2}\sqrt[3]{16}$

Answer the following. Simplify radicals if possible.

199. Find the value of 'a'.

$$\sqrt{150} = a\sqrt{6}$$

200. Find the value of 'a'.

$$\sqrt{128} = 2a\sqrt{2}$$

201. Find the value of 'a'.

$$\sqrt{96} = 4\sqrt{2a}$$

202. The two shorter sides of a right triangle are 8 cm and 2 cm. Using the Pythagorean Theorem $a^2 + b^2 = c^2$, find the length of the third side in simplest radical form.

203. The two legs of an isosceles right triangle are 5 cm. Using the Pythagorean Theorem $a^2 + b^2 = c^2$, find the length of the third side in simplest radical form.

204. Explain, using an example, how you simplify a radical using the multiplication of radicals method.

205. Explain, using an example, how you simplify a radical using pairs of prime factors of the radicand method.

Multiply and simplify if possible.

206. $\sqrt{18} \times \sqrt{12}$	207. $3\sqrt{20} \times 2\sqrt{5}$	208. $-5\sqrt{10} \times -2\sqrt{21}$
209. $2\sqrt{7} \times 3\sqrt{1} \times \sqrt{7}$	210. $-2(3\sqrt{6})(-\sqrt{8})$	211. $3\sqrt{7} \times 2\sqrt{6} \times -5\sqrt{2}$
212. $-2(3\sqrt{2})^3$	213. $(3\sqrt{5})^3(2\sqrt{2})^3$	214. $(\sqrt[3]{9})(\sqrt[3]{9})$
215. $\sqrt[3]{4} \times \sqrt[3]{8}$	216. $2\sqrt[3]{3} \times 5\sqrt[3]{18}$	217. $-\sqrt[3]{4} \times -3\sqrt[3]{12}$

Simplify.

218. Find the side length of a square with an area of $192\ m^2$.

219. Find the side length of a square with an area of $250\ cm^2$.

220. Find the area of a square with side lengths $2\sqrt{3}$ cm .

221. Find the area of a rectangle in simplest radical form if the dimensions are $\sqrt{12}$ cm , and $\sqrt{20}$ cm.

222. Find the area of a rectangle in simplest radical form if the dimensions are $\sqrt{108}$ mm and $\sqrt{175}$ mm.

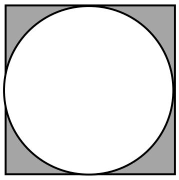
223. Calculate the exact area (radical) of a triangle that has base $\sqrt{14}$ mm and a height $\sqrt{28}$ mm.

224. Calculate the exact area (radical) of a triangle that has base $5\sqrt{10}$ m and a height $3\sqrt{30}$ m.

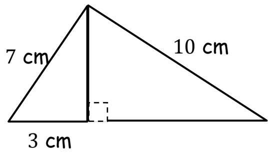
225. Find the length of a rectangle if its area is $6\sqrt{18}$ and its width is $3\sqrt{6}$

226. A rectangle has an area of $6\sqrt{15}$. Find possible side lengths that are mixed radicals.

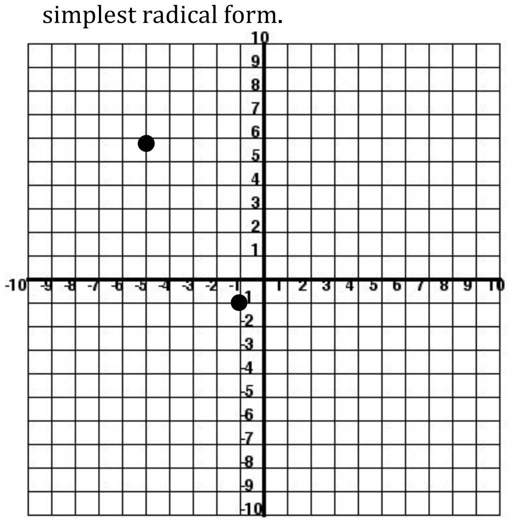
227. A circle of diameter $\sqrt{5}$ mm is inscribed in a square. Find the area of the square not covered by the circle. Answer to the nearest tenth.



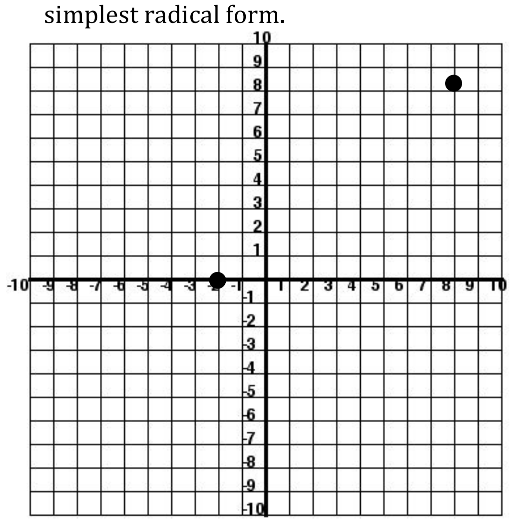
228. Find the area of the triangle below. Answer to the nearest tenth.



229. Find the distance between the two points in simplest radical form.



230. Find the distance between the two points in simplest radical form.



231. A 40 m ramp extends from a floating dock up to a parking lot, a horizontal distance of 30 m. How high is the parking lot above the dock? Answer in simplest radical form.

232. A fishing boat trolling in Haro Strait lets out 420 ft of fishing line. The lure at the end of the line is 100 ft behind the boat and the line starts 8 feet above the water. How deep is the lure?

What assumptions did you need to make to answer this question?

233. A biology student studying the root mass of conifers in British Columbia developed a formula to approximate the radius of the root mass. The formula uses the circumference of the tree trunk at ground level to calculate the radius of the roots.

$r = \sqrt{200C}$,
'r' is the radius of the root mass in metres.
C is circumference in metres.

Write the formula in simplest radical form.

234. Use the formula $r = \sqrt{200C}$ to calculate the radius of the root mass of a tree if the circumference is 1.8 m. Answer to the nearest tenth.

235. Does the formula work with a circumference in units other than metres? Explain why or why not.

236. Calculate the radius of the root mass if the circumference is 120 cm. Answer to the nearest tenth.

237. Calculate the circumference of a tree trunk at ground level if the root mass has a radius of 2.3 m.

238. Calculate the circumference of a tree trunk at ground level if the root mass has a radius of 145 cm.

239. The braking distance of Mr.J's farm truck can be used to calculate the speed the truck was travelling when it began braking. Below is the formula where 's' is the speed in km/h and 'd' is the distance required to stop in feet.

$$s = \sqrt{60d}$$

Calculate the speed his truck was travelling if it took 100 feet to stop. Answer to the nearest tenth.

240. Calculate the braking distance if Mr. J's truck was travelling at 50 km/h. Answer to the nearest tenth.

241. Challenge
Write $2\sqrt{5}$ as an entire radical.

242. Challenge
Write $5\sqrt{6}$ as an entire radical.

243. Challenge
Without using a calculator, arrange the following radicals in ascending order.
Show Work. $6\sqrt{2}, 3\sqrt{7}, 2\sqrt{17}, 4\sqrt{5}$

Writing Mixed Radicals as Entire Radicals.

Remember the process you used to simplify entire radicals → mixed radicals.

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

You will need to reverse the process…

Eg. Write $2\sqrt{5}$ as an entire radical.

$2 \times \sqrt{5}$	Convert the whole number,2, to a radical. 2 is equivalent to $\sqrt{4}$
$\sqrt{4} \times \sqrt{5}$	Multiply the radicands.
$= \sqrt{20}$	

Eg. Write $5\sqrt{6}$ as an entire radical.

$5 \times \sqrt{6}$	Convert the whole number,5, to a radical. 5 is equivalent to $\sqrt{25}$
$\sqrt{25} \times \sqrt{6}$	Multiply the radicands.
$= \sqrt{150}$	

Eg. Arrange in ascending order. $6\sqrt{2}, 3\sqrt{7}, 2\sqrt{17}, 4\sqrt{5}$

$6\sqrt{2} = \sqrt{36} \times \sqrt{2} = \sqrt{72}$
$3\sqrt{7} = \sqrt{9} \times \sqrt{7} = \sqrt{63}$
$2\sqrt{17} = \sqrt{4} \times \sqrt{17} = \sqrt{68}$
$4\sqrt{5} = \sqrt{16} \times \sqrt{5} = \sqrt{80}$

Ascending Order: $3\sqrt{7}, 2\sqrt{17}, 6\sqrt{2}, ,4\sqrt{5}$

Write as entire radicals.

244. $4\sqrt{3}$	245. $5\sqrt{3}$	246. $3\sqrt{10}$
247. $10\sqrt{3}$	248. $-4\sqrt{5}$	249. $-7\sqrt{2}$

Write as entire radicals.

250. $2\sqrt[3]{3}$

251. $4\sqrt[3]{2}$

252. $5\sqrt[3]{4}$

253. $3\sqrt[4]{2}$

254. $-2\sqrt[5]{3}$

255. $-3\sqrt[4]{4}$

256. $3\sqrt[4]{3}$

257. $10\sqrt[4]{2}$

258. $-4\sqrt[5]{5}$

259. Explain, in detail, how you could arrange a list of irrational numbers written in simplified radical form in ascending order without using a calculator.

Arrange in ascending order without using a calculator. Show Work.

260. $5, 4\sqrt{2}, 2\sqrt{6}, 3\sqrt{3}$

261. $4\sqrt{5}, 5\sqrt{3}, 2\sqrt{19}, 6\sqrt{2}, 3\sqrt{10}$

262. $3\sqrt{11}, 4\sqrt{5}, 7\sqrt{2}, 2\sqrt{21}, 6\sqrt{3}$

Mixed Practice

263. What sets of numbers does $2\sqrt{5}$ belong?

264. What sets of numbers does $\frac{12}{3}$ belong?

265. Write the number 9 in the following forms:

a) product of its primes________

b) as a radical__________

266. Write 720 as a product of its primes.

267. Explain how you could use the prime factors of 784 to find the square root. Then find the square root of 784.

268. Find the greatest common factor of the following sets of numbers.

a) 96, 224, 560

b) 140, 420, 560

269. Write 512 as a product of its primes. Use the factors to find $\sqrt[3]{512}$.

270. Use the pattern in the previous question to find $\sqrt[3]{a^9}$

271. Simplify the following.

a) $\sqrt{5} \times \sqrt{3}$

b) $-2\sqrt{7} \times 3\sqrt{5}$

c) $\sqrt[3]{10} \times \sqrt{2}$

272. Simplify the following.

a) $\sqrt{150}$

b) $-2\sqrt{180}$

c) $\sqrt[3]{192}$

273. Multiply and simplify the following.

$$\sqrt{12} \times 2\sqrt{3}$$

274. Multiply and simplify the following.

$$\sqrt{20} \times 2\sqrt{12}$$

275. The "living space" in Kai's tree fort is a perfect cube. The volume of the living space is 104 m^3. Find the area of carpet he will need to cover the floor. Answer to the nearest tenth.

276. A pizza just fits inside of a square box with an area of 625 cm^2. Find the area of the bottom of the box that is not covered by the pizza. Round to the nearest unit.

277. Find the perimeter of a square that has an area of 20 m^2. Answer as a mixed radical.

278. Without a calculator, arrange the following in descending order.

Show Work. $4\sqrt{5}, 3\sqrt{6}, 2\sqrt{10}, 5\sqrt{3}, 6\sqrt{2}$

ADDITIONAL MATERIAL

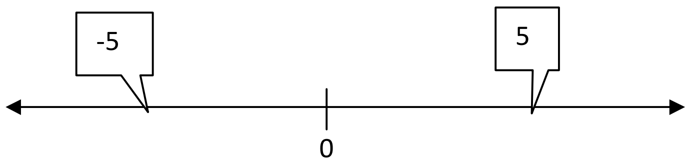
Absolute Value: $|x|$

The Absolute Value of a real number is its numerical value ignoring its sign.
Straight brackets around an expression indicate the absolute value function.

Eg. $|5|$ reads "the absolute value of five."
Eg. $|7-12|$ reads "the absolute value of seven minus twelve."

Absolute value is defined as the distance from zero on the number line.

Recall, distance cannot be a negative number. Both 5 and -5 are five units from zero.



Simplify the following.

1. $|-12| =$	2. $|7| =$	3. $|-2.54| =$

The absolute value symbol is a type of bracket. This means that operations inside the symbol must be performed first.

Eg. $|\mathbf{2-5}| \quad = |-3| \quad = 3$ Eg. $\mathbf{-2|7-12|} \quad = -2|-5| \quad = -2(5) \quad = -10$

Evaluate the following.

4. $|3+4-9|$	5. $\left|\frac{-7}{2}+\frac{2}{3}\right|$	6. $-|3(2-5)|$
7. $-5|3+7|$	8. $|2-7|-|5+3|$	9. $2|-9-2|-3|6-5|$

